FACULTY OF SCIENCE
Department Of Mathematics & Statistics

Introduction and Exponential functions

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MATH 1MM3 Winter 2023
Lecture 1

Intro

- Course page: **childsmath.ca**, then log in and pick the Math 1MM3 course.
- Contact info: babeia@mcmaster.ca
- Office hours: Hamilton Hall 417, We 12:20-1:20, Th 2:30-3:30.
- Textbook: *Applied Calculus for Business, Economics, and the Social and Life Sciences*, 11th edition by Laurence Hoffmann and Gerald Bradley, published by McGraw-Hill, in either bound, unbound or electronic version.
- 10 homework assignments throughout the term, 3 tests (multiple choice and in person), and the exam. Final exam date TBA.
- Course notes will be posted here: **angelicababei.com/math-1mm3-notes/**

Intro

1. Activities during class time: short recall questions, and short pair/individual exercises. These are here so that you can take a minute and digest all the new information better.
2. Occasional surveys, to be posted in announcements, to get feedback on how the course is running.

Power functions and exponents

Recall power functions: x^a } fixed
variable

Examples:

$x^4 = \underbrace{x \cdot x \cdot x \cdot x}_{4 \text{ times}}$

$\frac{1}{x^2} = x^{-2}$

$x^{2/3} = \sqrt[3]{x^2}$; $\sqrt{x} = x^{1/2}$

$x^0 = 1$

Combining certain power functions gives polynomials. Examples:

$p(x) = 1 + x + x^2 + 3x^3$

$p(x) = 2 - 3x$

$p(x) = 1 = x^0$

} polynomials have positive, integral powers

Today: exponential functions

Exponential functions grow very fast, faster than any polynomial or power function.

b^x } variable changes
base b fixed

Examples:

1. 2^x
2. $\left(\frac{1}{2}\right)^x$
3. $\left(\sqrt{3}\right)^x$

Exponential rules: Let $a, b > 0$, $a, b \neq 1$, $x, y \in \mathbb{R}$.

1. Equality: $b^x = b^y \iff x = y$! same base : b
2. Product: $b^x b^y = b^{x+y}$! mult → add exponents
3. Quotient: $\frac{b^x}{b^y} = b^{x-y}$! division → subtract exponents
4. Power: $(b^x)^y = b^{xy}$! power → multiply exponents
5. Multiplication: $(ab)^x = a^x b^x$
6. Division: $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Difference b/w ③ and ⑥ → $\left(\frac{2}{3}\right)^7 = \frac{2^7}{3^7}$

$$\frac{2^4}{2^6} = 2^{4-6} = 2^{-2}$$

Ex: Simplify $\frac{a^{-1}}{a^2} a^4 = a^?$

$$\frac{a^{-1}}{a^2} \overset{③}{=} a^{-1-2} = a^{-3}$$

$$= a^{-3} a^4 \overset{②}{=} a^{-3+4} = a^1 = a$$

Example 1

! Trick: get to the same base

Simplify

$$\left(\frac{(x+3)^5}{(x+3)^2}\right)^{-1}(x+3)^{-4}$$

$$\overset{(3)}{=}\left((x+3)^{5-2}\right)^{-1}(x+3)^{-4}$$

$$=\left((x+3)^{3}\right)^{-1}(x+3)^{-4}$$

$$\overset{(4)}{=}(x+3)^{3\cdot(-1)}(x+3)^{-4}$$

$$=(x+3)^{-3}(x+3)^{-4}\overset{(2)}{=}(x+3)^{-3-4}$$

$$=(x+3)^{-7}=\frac{1}{(x+3)^7}$$

Example 2

Simplify

$$\frac{3^{-2}6^{-1/2}}{27^{2/3}2^{1/2}}$$

Here, bases 3, $6 = 2 \cdot 3$

$27 = 3 \cdot 3 \cdot 3 = 3^3$

2

Bring to smallest bases, 3 and 2

$$= \frac{3^{-2}(2\cdot 3)^{-1/2}}{(3^3)^{2/3}\,2^{1/2}} \overset{(3)}{=} \frac{3^{-2}\,2^{-1/2}\,3^{-1/2}}{(3^3)^{2/3}\,2^{1/2}}$$

$$\overset{(4)}{=} \frac{3^{-2}2^{-1/2}3^{-1/2}}{3^{3\cdot\frac{2}{3}}\,2^{1/2}} = \frac{3^{-2}2^{-1/2}3^{-1/2}}{3^2\,2^{1/2}}$$

$$= 3^{-2-1/2-2}\,2^{-1/2-1/2} = 3^{-4.5}\,2^{-1}$$

Take a minute to digest!

Take a minute to write as much as you can remember from the past 15 minutes.

More practice: Appendix A.1 + Hw 1.