Last time: Extreme value theorem on a closed interval.

Today: More general scenarios.

Idea: Rough sketch of graph

Need:
- Domain or interval of interest.
- Intervals of increase/decrease
- Critical points and relative min and max

→ first derivative

- values or limits at endpoints

For more accurate graphs
- intervals of concave up/down, inflection points

→ second derivative (section 3.2)

- intercepts (intersections with x- and y-axis) Section 1.2.

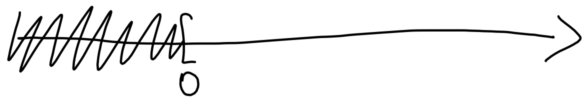
Example: In an electronics firm, when x thousand employees work there, the profit is

$$P(x) = \ln(4x+1) + 3x - x^2$$

(million dollars).

How many employees maximizes profit? What is the max profit?

Solution: Need $x \geq 0$. This works out for $\ln(4x+1)$ to be defined, because then $4x+1 > 0$ when $x \geq 0$.



This is an infinite interval, cannot use the Extreme Value Theorem.

Next: $P'(x)$

$$P'(x) = \left(\ln(4x+1) + 3x - x^2\right)'$$

$$\boxed{[\ln(u(x))]' = \frac{u'(x)}{u(x)}}$$

$$= (4x+1)' \cdot \frac{1}{4x+1} + 3 - 2x$$

$$= \frac{4}{4x+1} + 3 - 2x$$

$$= \frac{4 + 3(4x+1) - 2x(4x+1)}{4x+1}$$

$$= \frac{4 + 12x + 3 - 8x^2 - 2x}{4x+1}$$

$$P'(x) = \frac{-8x^2 + 10x + 7}{4x+1}$$

Find critical points.

$P'(x)$ DNE : $4x+1=0$

$x=-\frac{1}{4}$

$x=-\frac{1}{4}$ not in the domain! Ignore.

$P'(x)=0$.

$$\frac{-8x^2+10x+7}{4x+1}=0$$

$$\frac{-8x^2+10x+7}{\cancel{4x+1}}\cdot\cancel{(4x+1)}=0\cdot(4x+1)$$

$$-8x^2+10x+7=0$$

Quad formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-10\pm\sqrt{(10)^2-4\cdot(-8)\cdot 7}}{2\cdot(-8)}$$

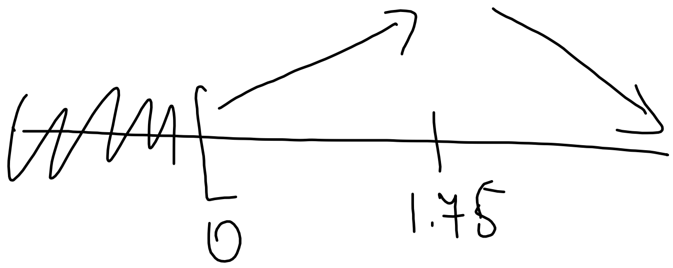
$$x=\frac{-10\pm\sqrt{100+224}}{-16}$$

$$x=\frac{-10\pm 18}{-16}$$

So $x=\frac{8}{-16}=-\frac{1}{2}$ } not in domain. Ignore it.

$$x=\frac{-28}{-16}=\frac{7}{4}=1.75$$

One critical point: $x=1.75$.



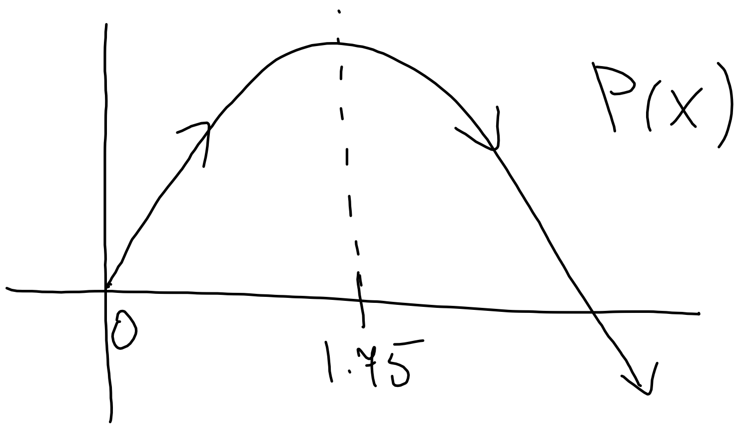
Intervals of increase/decrease:

try $x = 0$ or 1.

$$P'(0) = \frac{7 + 10\cdot 0 - 8\cdot 0^2}{4\cdot 0 + 1} = 7 > 0$$

$$P'(2) = \frac{7 + 10\cdot 2 - 8\cdot 2^2}{4\cdot 2 + 1}$$

$$= \frac{7 + 20 - 32}{9} < 0$$



So, $x = 1.75$ gives absolute max.

Max profit is $P(1.75) \approx 4.27$ (million dollars)

Applications:

1. Learning curve
2. Logistic curve

• Learning curve

$$Q(t) = B - Ae^{-kt}$$

productivity at time t $\quad A, B, k > 0$ depend on context

Example: A student's score on a test is a function of the number of hours studied

$$S(t) = 100 - 80e^{-0.2599t}$$

(a) How many hours do they have to study to get 50%?

Solve for t:

$$50 = 100 - 80e^{-0.2599t}$$