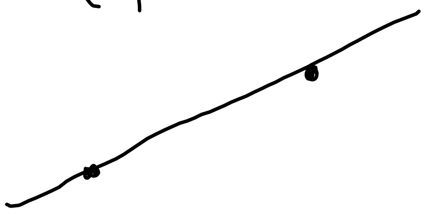
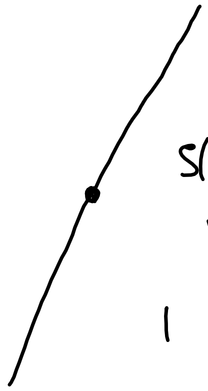
Recall: The equation of a line

2 points: 1 line





slope + point: 1 line

Chapter 1.3: Slope m, point (x_0, y_0),
point-slope form

$$y - y_0 = m(x - x_0)$$

(e.g.: line with slope $\frac{1}{3}$ thru $(3,5)$)

$$y - 5 = \frac{1}{3}(x - 3)$$

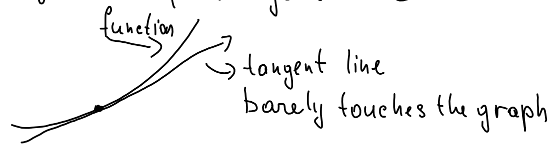
To get in slope-intercept,

$$y = \frac{1}{3}(x-3)+5$$

$$y = \frac{1}{3}x - \frac{1}{3}\cdot 3 + 5$$

$$y = \frac{1}{3}x + 4$$

Equation of tangent line



To get the equation:

① point $(a, f(a))$ on the graph

② slope of the tangent line at $x = a$ is the derivative $f'(a)$

This gives

$$\boxed{y - f(a) = f'(a)(x - a)}$$

Example: Find the equation of the tangent line to the graph of $f(x) = x - \ln(\sqrt{x})$ at the point $x = 1$.

Solution: The point has $x = 1$, so the y-coordinate is

$$f(1) = 1 - \ln(\sqrt{1}) = 1 - \ln(1)$$
$$= 1 - 0 = 1$$

Point is $(1, 1)$.

The slope is $f'(1)$. Need $f'(x)$:

$$(x - \ln(\sqrt{x}))' = (x)' - (\ln(\sqrt{x}))'$$
$$= 1 - (\sqrt{x})' \cdot \frac{1}{\sqrt{x}}$$
$$= 1 - \frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{x}} = 1 - \frac{1}{2x}$$

So $f'(x) = 1 - \frac{1}{2x}$, and

$$f'(1) = 1 - \frac{1}{2 \cdot 1} = \frac{1}{2}$$

So $y - 1 = \frac{1}{2}(x - 1)$

or $y = \frac{1}{2}(x-1) + 1$

$$y = \frac{1}{2}x - \frac{1}{2} + 1$$

$$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$$

Review of 2.5: Marginal analysis

Cost function $C(x)$

$x = \#$ units $\quad p(x) =$ price depending on units (per unit)

Revenue $R(x) = \#\text{units} * \text{price}$
$= x \cdot p(x)$

Profit $P(x) = \text{revenue} - \text{cost}$
$= R(x) - C(x)$
$= x \cdot p(x) - C(x)$

Marginal cost $C'(x)$
— " — revenue $R'(x)$
— " — profit $P'(x)$

The marginal functions estimate the additional

cost $C'(x_0) \approx C(x_0+1) - C(x_0)$

revenue $R'(x_0) \approx R(x_0+1) - R(x_0)$

profit $P'(x_0) \approx P(x_0+1) - P(x_0)$

when we increase production by 1 from x_0 to x_0+1. (if x_0 is large enough).

Example: A company manufacturing fuel tanks for automobiles. The cost of manufacturing the x^{th} fuel tank is described by $C(x)$. If the marginal cost of manufacturing the 25^{th} fuel tank is -20, and $C(25) = 1150$, estimate the cost when we increase production from 25 to 26.

Solution: Marginal cost

$$C'(x_0) \approx C(x_0+1) - C(x_0)$$

so $C(x_0+1) \approx C(x_0) + C'(x_0)$

Here, $C(26) \approx 1150 + (-20)$
≈ 1130

Generally, if we increase x_0 by Δx (here, Δx is small relative to x_0),

$$f'(x_0)\Delta x \approx f(x_0+\Delta x) - f(x_0)$$

Previously, $\Delta x = 1$

So $$\underbrace{f(x_0+\Delta x) \approx f(x_0) + f'(x_0)\Delta x}_{\text{linear approximation.}}$$