Linear approximation

$$f(\underbrace{x_0+\Delta x}_{\text{small}}) \approx f(x_0)+f'(x_0)\Delta x$$

Example: The demand for a product at a given price p,

$$D(p) = 200 - 0.2p^2 \ (\# \text{ of units})$$

(a) What is the revenue when the price is \$10?

Revenue = #units * price

Here,

$$R(p) = D(p)\cdot p$$
$$= (200 - 0.2p^2)\cdot p$$
$$= 200p - 0.2p^3$$

At $p = \$10$, revenue is

$$R(10) = 200\cdot 10 - 0.2\cdot 10^3 = 2000 - 200$$
$$= 1800$$

(b) How much additional revenue do we get if we increase the price from 10 to 11 \$?

Soln: We can get an exact answer: find revenue at $p=11$, then revenue at $p=10$, then subtract.

$R(10) = 1800$ (from part (a))

$R(11) = 200\cdot 11 - 0.2\cdot 11^3 = 1933.80$

So additional revenue is

$$R(11) - R(10) = 1933.80 - 1800$$
$$= \boxed{133.80}$$

(c) Estimate the additional revenue using marginal analysis:

(generally, $R(x_0+1) \approx R(x_0) + R'(x_0)$)

Here,

$$R'(p) \approx R(p+1) - R(p)$$
$$R'(10) \approx R(11) - R(10)$$

To get marginal revenue, find $R'(p)$.

$$R'(p) = (200p - 0.2p^3)'$$
$$= 200 - 0.2\cdot 3\cdot p^2$$
$$= 200 - 0.6p^2$$

At $p = 10$,

$$R'(10) = 200 - 0.6\cdot 10^2$$
$$= 200 - 60 = \boxed{140}$$

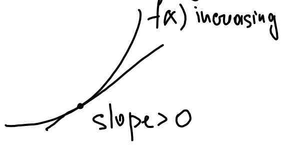
Caveat: Answers in (b) (133.80) and (c) (140) should be close.

However, to improve accuracy, can make Δx smaller.

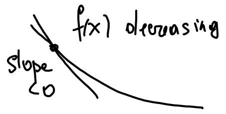
Next: Maximize profit? Minimize cost?

Review of 3.1:

Increasing/decreasing functions



f(x) increasing at $x=c$ if $f'(c)>0$



f(x) decreasing at $x=c$ if $f'(c)<0$

"Critical point": if $x=c$ is IN THE DOMAIN of $f(x)$ and $f'(c)$ doesn't exist OR $f'(c)=0$.

Ex: Find the critical points, if any, of $f(x)=xe^{2x}$.

Solution: Critical points: need $f'(x)$.

$$f'(x)=(xe^{2x})' \xrightarrow[\text{rule}]{\text{product}} (x)'\cdot e^{2x}+x\cdot(e^{2x})'$$

x $\quad 1$

e^{2x} $\quad (2x)'\cdot e^{2x}=2e^{2x}$

chain rule

$$= 1\cdot e^{2x}+x\cdot 2e^{2x}$$

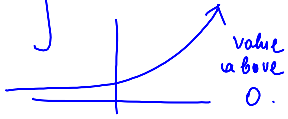
$$=(1+2x)e^{2x}$$

So $f'(x)=(1+2x)e^{2x}$

$f'(x)$ is defined everywhere.

Only thing we need now is $f'(x)=0$:

$(1+2x)e^{2x}=0$ $\quad$ e^{2x} never 0

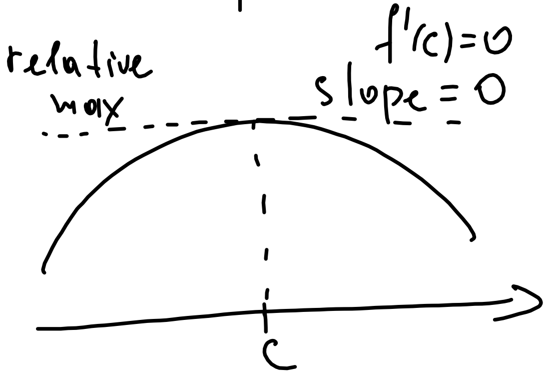


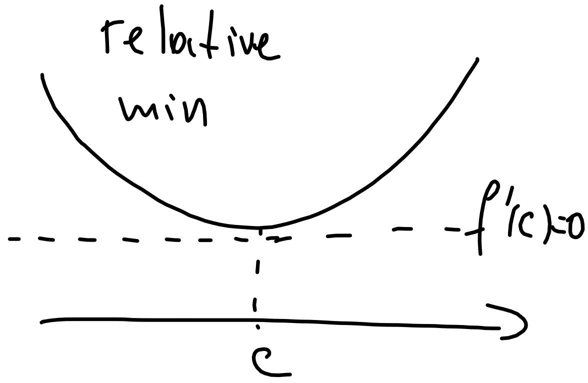
so $1+2x=0$

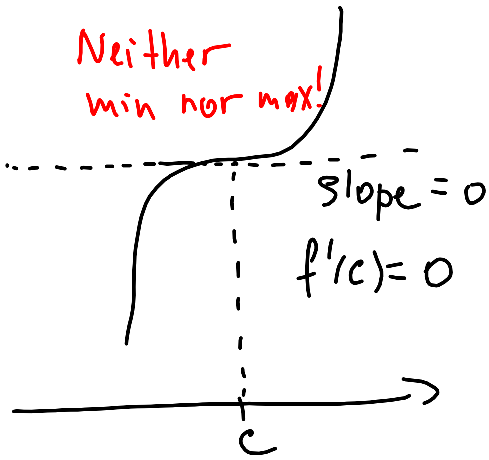
$x=-\frac{1}{2}$.

Critical point at $\boxed{x=-\frac{1}{2}}$

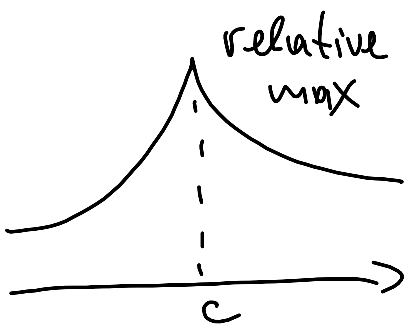
Critical points are important because min and max values have to be at these points.

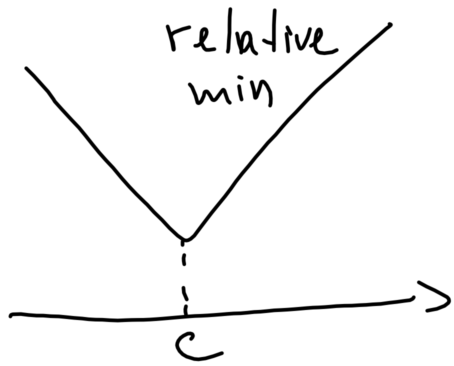






$f'(c)$ not defined



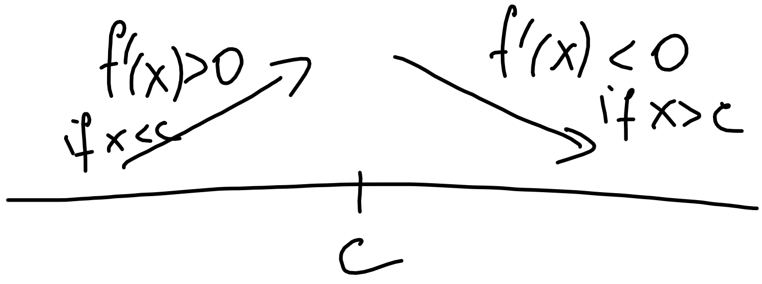


Caution: Not all critical points are minima or maxima.

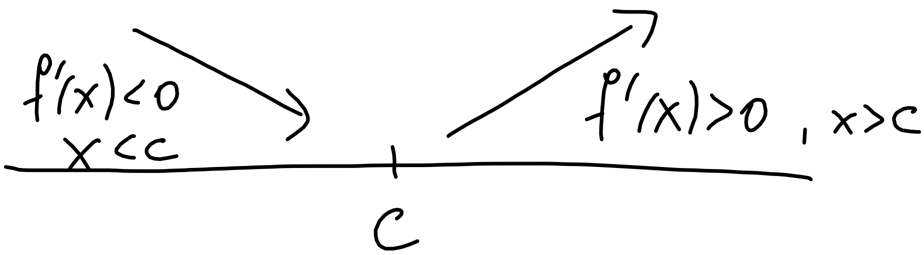
To classify critical points, we can perform 2 tests:

① The first derivative test

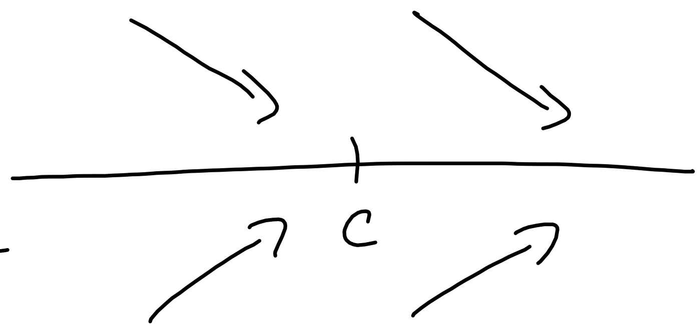
if $x = c$ is a critical point, then $(c, f(c))$ is a relative max if



relative min if



neither min nor max if



or

$f'(x)$ has the same sign before and after $x = c$.