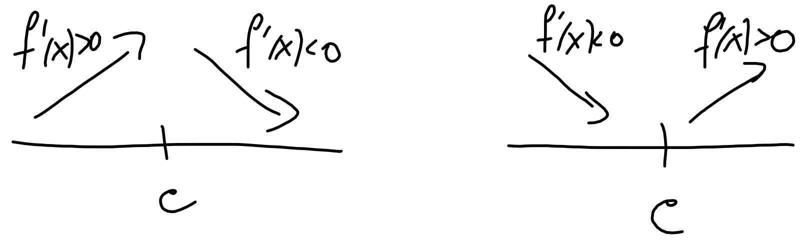
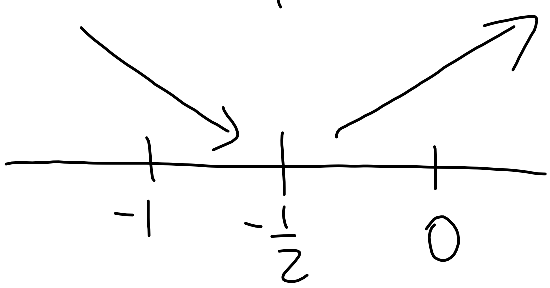
- First derivative test



<u>Example:</u> $f(x) = xe^{2x}$

Found $x = -\frac{1}{2}$ critical point, $f'(x) = (1+2x)e^{2x}$. Classify the critical point.



$$f'(-1) = (1+2\cdot(-1))e^{2\cdot(-1)}$$
$$= (1-2)e^{-2} = -e^{-2} < 0$$

$$f(0) = (1+2\cdot 0)e^{2\cdot 0} =$$
$$= 1\cdot e^{0} = 1 > 0$$

So $x = -\frac{1}{2}$ gives a relative minimum

• The second derivative test

If $x=c$ is a critical value and $f'(c)=0$

then $(c, f(c))$ is a

relative min if $f''(c)>0$

Why? If $f''(c)>0$, graph is concave up

relative max if $f''(c)<0$

Why? $f''(c)<0$, concave down

Ex: Use the second derivative test to classify $x=-\frac{1}{2}$ in $f(x)=xe^{2x}$

$f'(x)=(1+2x)e^{2x}$

Soln: Need to check $f'(-\frac{1}{2})=0$:

$f'(-\frac{1}{2})=(1-2\cdot\frac{1}{2})e^{2\cdot(-\frac{1}{2})}=0$

So we can use the test!

Next, find $f''(x)$:

$((1+2x)e^{2x})'$

$=(1+2x)'\cdot e^{2x}+(1+2x)\cdot(e^{2x})'$

$=2e^{2x}+(1+2x)\cdot(2x)'\cdot e^{2x}$

$=2e^{2x}+(1+2x)\cdot 2e^{2x}$

Next, find value at $x=-\frac{1}{2}$

$x \xrightarrow{2} 2x \xrightarrow{e^{2x}} e^{2x}$ chain rule

$f''(-\frac{1}{2})=\boxed{2\cdot e^{2\cdot(-\frac{1}{2})}}+\underbrace{\cancel{(1+2\cdot(-\frac{1}{2}))}}_{0}\cdot 2e^{2\cdot(-\frac{1}{2})}$

$=2e^{-1}>0$

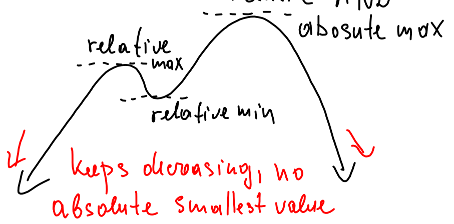
concave up

So $x=-\frac{1}{2}$ gives a min.

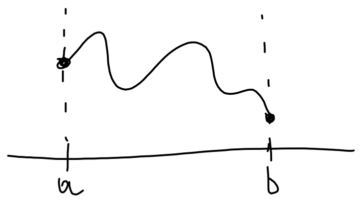
If $f''(c)=0$, we cannot tell, use first derivative test.

What about absolute minima and maxima?

Tricky: Not all functions have absolute largest or smallest values



However, smallest and largest values always exist if function is on a closed interval.



Extreme value property

(Chapter 3.4)

If $f(x)$ is continuous on the CLOSED interval $a \le x \le b$, it has both absolute min and max.

In such cases, to find absolute min and max, look at the values of $f(x)$ at the endpoints, and at the critical points

Ex: Let $f(x) = 9e^{x^2-10x}$

Find the largest and smallest values of $f(x)$ on the interval $0 \le x \le 6$

Solution: $f(x)$ defined everywhere, and it is continuous.

Critical points: find $f'(x)$

$$\left(9e^{x^2-10x}\right)' = 9 \cdot (2x-10)\, e^{x^2-10x}$$

$$x \xrightarrow{2x-10} x^2-10x \xrightarrow{e^{x^2-10x}} e^{x^2-10x}$$

$$f'(x) = 9(2x-10)\,e^{x^2-10x}$$

Defined everywhere. Next: find $f'(x)=0$

$$\cancel{9}(2x-10)\cancel{e^{x^2-10x}} = 0$$

$$2x-10=0$$

$\boxed{x=5}$ is on $0 \le x \le 6$

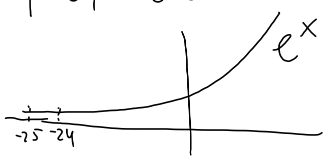
x	0	5	6
$f(x)$	9	$9e^{-25}$	$9e^{-24}$

not $f'(x)$!!

$$f(0) = 9e^0 = 9$$

$$f(5) = 9e^{5^2-10\cdot 5} = 9e^{-25}$$

$$f(6) = 9e^{6^2-10\cdot 6} = 9e^{-24}$$



Largest value at $x=0$: value 9

Smallest at $x=5$: value $9e^{-25} \approx 0$