FACULTY OF SCIENCE
Department Of Mathematics & Statistics

(Moar) Logarithmic Functions

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Lecture 4

Introductory examples

Recall:

$$\boxed{v = \log_b a \iff b^v = a}$$

1. Take a minute to solve for x the following equation. After a minute check your solution with a classmate.

$$\log_2 x = 4$$

Here, $v = 4$, $b = 2$, $a = x$

Then $x = 2^4 = 16$

2. This one we solve together: earlier problem to solve for t

$$e^{0.05t} = 2.$$

We saw that $0 < t < 20$

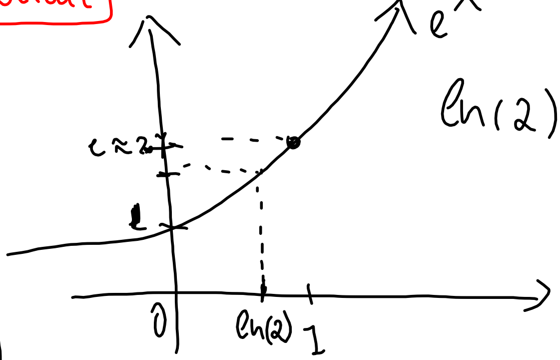
$$\boxed{v = \ln a \iff e^v = a}$$

Here, $v = 0.05t$, $a = 2$

so $0.05t = \ln(2)$

$\ln(2)$ is the power that we need to raise e to, to get 2.

Detour



$\ln(2) \approx 0.693147...$

So $0.05t = \ln(2)$

$$\boxed{t = \frac{\ln(2)}{0.05} \approx 13.86}$$

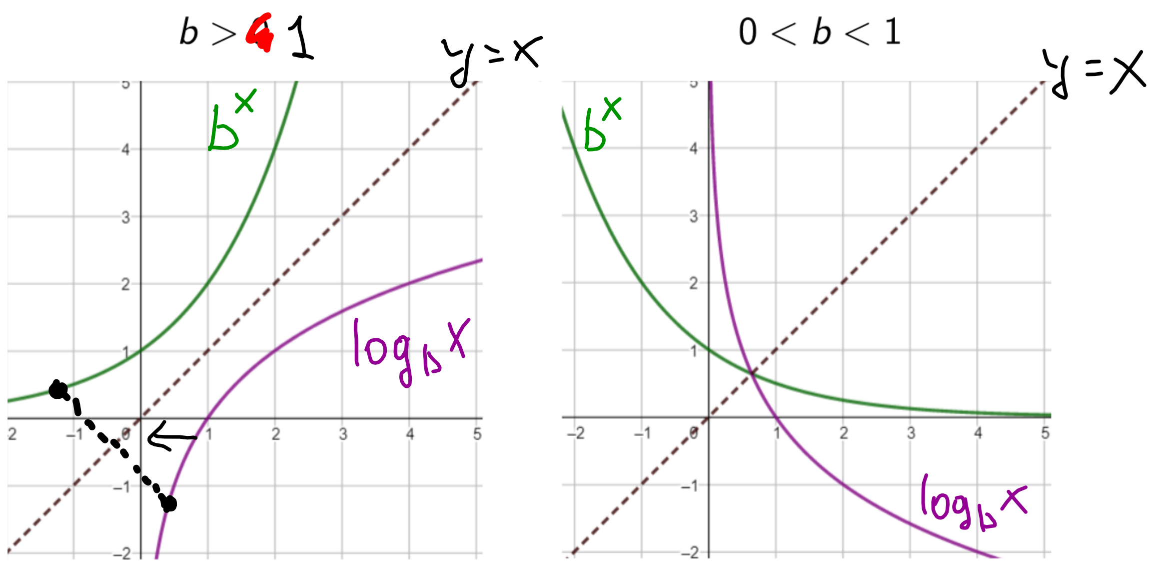
Logarithmic Functions

Definition. If $\boxed{x \text{ is a positive number}}$, then the **logarithm** of x to the base b ($b > 0, b \neq 1$), denoted by $\log_b x$, is the number y such that $b^y = x$. We write

$$y = \log_b x \quad \text{or} \quad f(x) = \log_b x.$$

Domain is $x > 0$

Graphs of Logarithmic Functions: 2 cases



The graph of $y = \log_b x$ is the reflection (mirror image) of the graph of $y = b^x$ about the line $y = x$.

<u>Case 1:</u> $b > 1$

$$\lim_{x \to 0^+} \log_b x = -\infty$$

$$\lim_{x \to +\infty} \log_b x = +\infty$$

<u>Case 2:</u> $0 < b < 1$

$$\lim_{x \to 0^+} \log_b x = +\infty$$

$$\lim_{x \to +\infty} \log_b x = -\infty$$

There is a vertical asymptote at $x = 0$.

Take a minute to digest!

Take a minute to write as much as you can remember from the past 20 minutes.

Logarithm rules (mirror exponential rules)

Let $u, v > 0, b > 0, b \neq 1$.

- $\log_b(1) = 0$ and $\log_b b = 1$

Why? $\log_b(1) = v$ then $b^v = 1$. So $v = 0$.

$\log_b(b) = v$ then $b^v = b$, so $v = 1$.

- Product:

$$\log_b(uv) = \log_b u + \log_b v.$$

- Quotient:

$$\log_b\left(\frac{u}{v}\right) = \log_b u - \log_b v.$$

- Power: if r is a real number, then

$$\log_b(u^r) = r\log_b(u).$$

Why? $\log_b(u^r) = \log_b(\underbrace{u \cdot u \cdot \ldots \cdot u}_{r \text{ times}})$

product

$$= \underbrace{\log_b u + \log_b u + \ldots + \log_b u}_{r \text{ times}}$$

$$= r \log_b u$$

Examples

Example 1. Write $\log_5 36$ in terms of $\log_5 2$ and $\log_5 3$.

$$\log_5(36) = \log_5(4 \cdot 9)$$

$$\overset{\text{Product}}{=} \log_5 4 + \log_5 9 = \log_5(2^2) + \log_5(3^2)$$

$$\overset{\text{power}}{=} 2\log_5 2 + 2\log_5 3$$

Example 2. Write $\log_2\left(\frac{y^3}{x^4}\right)$ in terms of $\log_2 x$ and $\log_2 y$.

$$\log_2\left(\frac{y^3}{x^4}\right) \overset{\text{quotient}}{=} \log_2 y^3 - \log_2 x^4$$

$$\overset{\text{power}}{=} 3\log_2 y - 4\log_2 x$$

Logarithm rules (continued)

Important for solving equations.

Let $u, v > 0, b > 0, b \neq 1$.

- Equality:

$$\log_b u = \log_b v \iff u = v. \Big\} \text{same base } b!$$

- Inversion:

$$\underbrace{\log_b(b^u) = u}_{\text{log cancels out exponentiation}} \quad \text{and} \quad \underbrace{b^{\log_b u} = u.}_{\text{exponentiation cancels out the log}}$$

Why? $\log_b(b^u) \overset{\text{power}}{=} u \log_b b \overset{\text{first rule}}{=} u \cdot 1$

Some examples:

$7^x = 8$, then $\underbrace{\log_7(7^x) = \log_7(8)}$

take $\log_7$ on both sides

$$\log_7(7^x) = x = \log_7 8$$

Solving logarithmic equations

Use equality or inversion

Exercise 1. Solve for x the equation

$$\ln(x) + \ln\left(\frac{x-6}{4x}\right) = \ln 2.$$

Bring to only one term on each side.

product

$$\ln\left(\cancel{x} \cdot \frac{x-6}{4\cancel{x}}\right) = \ln 2$$

$$\ln\left(\frac{x-6}{4}\right) = \ln 2$$

base e on both sides, use equality

$$\frac{x-6}{4} = 2$$

$$x - 6 = 2 \cdot 4 = 8$$

$$\boxed{x = 8 + 6 = 14}$$

Solving logarithmic equations (cont.)

Try using inversion!

Exercise 2. Solve for x the equation

$$3^{5x-1} = 12.$$