Learning curve

$$Q(t) = B - Ae^{-kt}$$

Example: Score on a test in terms of time studied

$$S(t) = 100 - 80e^{-0.2599t}$$

(a) To get score of 50:

Solve for t

$$50 = 100 - 80e^{-0.2599t}$$

$$80e^{-0.2599t} = 100 - 50 = 50$$

$$e^{-0.2599t} = \frac{50}{80} = \frac{5}{8}$$

Take ln of both sides

$$-0.2599t = \ln\left(\frac{5}{8}\right)$$

$$t = \frac{\ln\left(\frac{5}{8}\right)}{-0.2599} \approx 1.8084 \text{ (hours)}$$

How many hours to get 90%?

$$90 = 100 - 80e^{-0.2599t}$$

$$80e^{-0.2599t} = 100 - 90 = 10$$

$$\ln\left(e^{-0.2599t}\right) = \ln\left(\frac{10}{80}\right) = \ln\left(\frac{1}{8}\right)$$

$$-0.2599t = \ln\left(\frac{1}{8}\right)$$

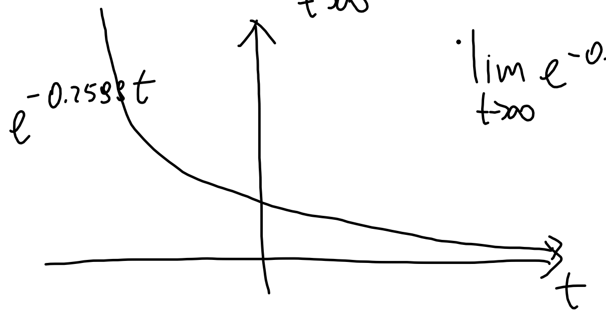
$$t = \frac{\ln\left(\frac{1}{8}\right)}{-0.2599} \approx 8.0009$$

(b) What score can the student get eventually? Eventually = take $t \to \infty$

$$\lim_{t\to\infty} \left(100 - 80e^{-0.2599t}\right)$$

$$= 100 - \lim_{t\to\infty} 80e^{-0.2599t}$$

$$= 100 - 80 \lim_{t\to\infty} e^{-0.2599t}$$

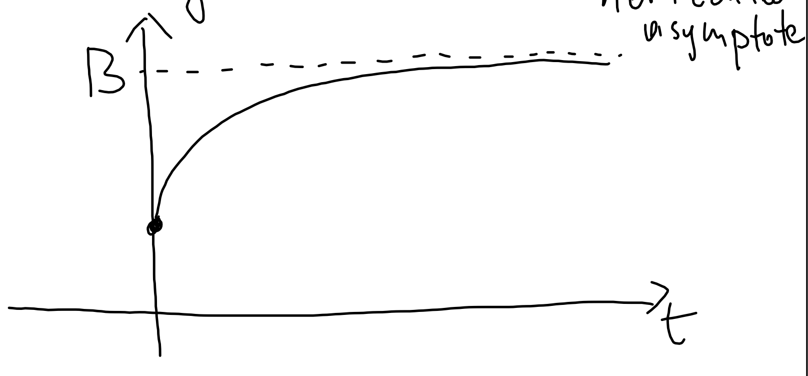


$\cdot \lim_{t\to\infty} e^{-0.2599t} = 0$

$$= 100 - 80 \cdot 0 = 100$$

$Q(t) = B - Ae^{-kt}$, $A, B, k > 0$

The shape of this learning curve is



B is the "learning capacity"

• Logistic curve

$$Q(t) = \frac{B}{1 + A e^{-B \cdot k \cdot t}}$$

$A, B, k > 0$ constants

Fast, but limited growth
(e.g. population, epidemic models)

Ex: It is estimated that t years from now, the population of a certain country will be

$$P(t) = \frac{20}{2 + 3e^{-0.06t}} \text{ (million)}$$

$$= \frac{10}{1 + \frac{3}{2} e^{-0.06t}}$$

(a) What is the current population?

Current : time $t = 0$

$$P(0) = \frac{20}{2+3e^{-0.06 \cdot 0}} = \frac{20}{2+3e^{0}} = \frac{20}{2+3 \cdot 1}$$

$$= 4 \text{ (million)}$$

(b) What will the population be in 50 years? So $t = 50$

$$P(50) = \frac{20}{2+3e^{-0.06 \cdot 50}} = 9.3051$$

(million)

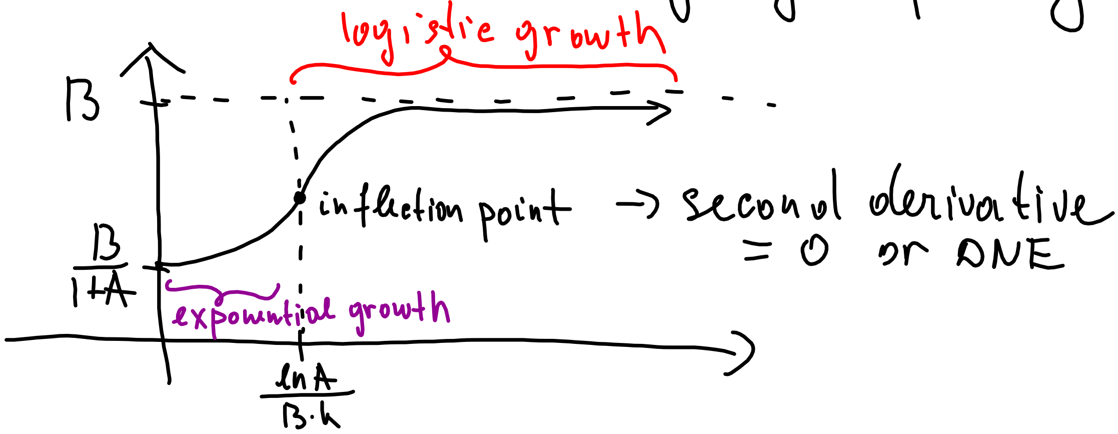
(c) What will happen to the population in the long run?

$$\lim_{t\to\infty} \frac{20}{2+3e^{-0.06t}} = \frac{20}{2+3\lim_{t\to\infty} e^{-0.06t}}$$

$$= \frac{20}{2+3\cdot 0} = 10 \text{ (million)}$$

Generally, $\lim_{t\to\infty} \frac{B}{1+Ae^{-kBt}} = \frac{B}{1+A\cdot 0} = B$

B is the "carrying capacity"



Chapter 5 : Integration

§5.1 Antiderivatives; indefinite integration

Qn: If you know the rate at which a population grows, can you determine the future population?

Rephrase: If you know the rate of change ($f'(x)$ or $\frac{df}{dx}$) of a function $f(x)$, can you find $f(x)$?

We go backwards: know the derivative, find the function

So far: $f(x) \longrightarrow f'(x)$

Now: $? \longleftarrow f'(x)$

$F(x)$ is the antiderivative of $f(x)$ if

$$F'(x) = f(x)$$

Examples: Consider the functions

ⓐ $\frac{1}{3}x^3 + 2x^2 + 3$

ⓑ $x^3 + x^2 + x + 1$

ⓒ $x^3 + x^2 + x - 6$

Which one of them is an antiderivative of $f(x) = 3x^2 + 2x + 1$

Soln: Compute the derivatives

ⓐ $\left(\frac{1}{3}x^3 + 2x^2 + 3\right)' = 3 \cdot \frac{1}{3}x^2 + 2 \cdot 2x + 0 = x^2 + 4x$