$F(x)$ is the antiderivative of $f(x)$ if $F'(x) = f(x)$

E.g: Which of the following is an antiderivative for $f(x) = 3x^2+2x+1$

(a) $\frac{1}{3}x^3 + 2x^2 + 3$

(b) $x^3 + x^2 + x + 1$

(c) $x^3 + x^2 + x - 6$

Soln: Take the derivative

(a) $\left(\frac{1}{3}x^3 + 2x^2 + 3\right)' = 3\cdot\frac{1}{3}x^2 + 2\cdot 2x + 0$
$= x^2 + 4x$ ✗

(b) $(x^3+x^2+x+1)' = 3\cdot x^2 + 2x + 1 + 0$
$= 3x^2 + 2x + 1$ ✓

(c) $(x^3+x^2+x-6) = 3x^2+2x+1$ ✓

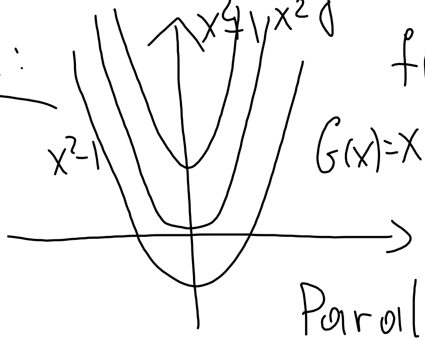
Note: Both (b) and (c) are antiderivatives of $f(x)$

In general, a function has infinitely many antiderivatives

$$G(x) = F(x) + C,$$

C is any constant

Ex:



$f(x) = 2x$

$G(x) = x^2 + C$
antiderivative

Parallel graphs

Notation:

$$\underbrace{\underbrace{\int}_{\text{integral symbol}} \overbrace{f(x)}^{\text{integrand}} \underbrace{dx}_{\text{variable of integration}} = \overbrace{F(x)}^{\text{antiderivative}} + \underbrace{C}_{\text{constant}}}_{\text{Indefinite integral}}$$

Ex: $\int 1\,dx = ?$

Which $F(x)$ gives $F'(x) = 1$?

$$F(x) = x \;!$$

$$\int 1\,dx = x + C$$

General rule:

$$\int k\,dx = kx + C$$

k some constant

- $\int \frac{1}{x}\,dx = ?$

$F(x) = \ln(x)$ gives

$F'(x) = \frac{1}{x}$ (To allow $x<0$, put absolute vals in $\ln(x)$)

Rule: $\int \frac{1}{x}dx = \int x^{-1}dx = \ln(|x|) + C$, $x \neq 0$

- $\int x^2 dx = ?$

Try $(x^3)' = 3x^2$

Divide by 3: $\left(\frac{1}{3}x^3\right)' = \cancel{3}\cdot\frac{1}{\cancel{3}}\cdot x^2$ ✓

General:

$$\int x^a dx = \frac{1}{a+1}x^{a+1} + C$$

$\boxed{a \neq -1}$

- $\int e^x dx = e^x + C$

General: $\int e^{kx}dx = \frac{1}{k}e^{kx} + C$, $k \neq 0$ constant

Ex:

① $\int 5\,dx = 5x + C$

② $\int e^{-2x}\,dx = -\frac{1}{2}e^{-2x} + C$

③ $\int x^{23}\,dx = \frac{1}{24}x^{24} + C$

④ $\int \frac{1}{\sqrt[3]{x}}\,dx = \int x^{-1/3}\,dx$

$= \frac{1}{-\frac{1}{3}+1}x^{-1/3+1} + C$

$= \frac{1}{\frac{2}{3}}x^{2/3} + C = \frac{3}{2}\sqrt[3]{x^2} + C$

Constant multiple:

$$\int k f(x)\,dx = k\int f(x)\,dx$$

Ex: $\int 6x\,dx = 6\int x\,dx$

$= 6\cdot\frac{1}{2}x^2 + C = 3x^2 + C$

Addition and subtraction

$$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int (f(x)-g(x))\,dx = \int f(x)\,dx - \int g(x)\,dx$$

E.g: $\int (x^2+3x-1)\,dx$

$= \int x^2\,dx + 3\int x\,dx - \int 1\,dx$

$= \frac{1}{3}x^3 + C_1 + 3\left(\frac{1}{2}x^2 + C_2\right)$

$- (x + C_3)$

$= \frac{1}{3}x^3 + \frac{3}{2}x^2 - x + C$

More examples:

① $\int 3e^x\,dx = 3\int e^x\,dx$

$= 3\cdot e^x + C$

② $\int \left(\frac{1}{2y} - \frac{2}{y^2} + \frac{3}{\sqrt{y}}\right)dy$

$= ?$