FACULTY OF SCIENCE
Department Of Mathematics & Statistics

# Section 5.2: Integration by substitution.

Angelica Babei

MATH 1MM3 Winter 2023
Lecture 17

## Example

$$\int \frac{3x^2+5}{x^3+5x+1}\,dx =$$

**Solution.** Let $u = x^3+5x+1$, then $\dfrac{du}{dx} = 3x^2+5$. $\rightsquigarrow du = \boxed{(3x^2+5)dx}$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$\shortparallel$$

$$\int \frac{1}{u}\,du$$

$$= \ln(|x^3+5x+1|) + C$$

## Dealing with extra constants.

$$\int \frac{\boxed{x}}{x^2+1}\boxed{dx} =$$

**Solution.**

Let $u = x^2+1$

$$\frac{du}{dx} = 2x \rightsquigarrow du = 2\boxed{x\,dx}$$

$$\frac{1}{2}du = x\,dx$$

$$= \int \frac{\frac{1}{2}du}{u} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(|x^2+1|) + C \underset{x^2+1>0}{=} \frac{1}{2}\ln(x^2+1) + C$$

## Example

$$\int x^3(36-22x^4)^2\,dx =$$

**Solution.**

Let $u = 36-22x^4 \rightsquigarrow \frac{du}{dx} = -22\cdot 4\cdot x^3 = -88x^3$

(Alternative: $\frac{du}{-88x^3} = dx$) $\frac{1}{-88}du = x^3dx$

$$= \int u^2\left(-\frac{1}{88}du\right) = -\frac{1}{88}\int u^2du = -\frac{1}{88}\cdot\frac{1}{3}u^3 + C$$

$$= -\frac{1}{264}(36-22x^4)^3 + C$$

Additional practice problems for integration by $u$-substitution (simpler examples).
Ex. 1, 7, 9, 11, 13, 15, 17 in Section 5.2.

## Take a minute to digest!

Take a minute to jot down what you can remember from these examples. For example, what did we look for that indicated that the problem was solvable by $u$-substitution?

## More involved examples I

Sometimes, it's harder to recognize the $u'(x)$ in the integrand. If the integral is still not solvable by basic formulas, try to look for a composition of functions (a function with layers) in the integral, and try to pick $u(x)$ as the middle layer.

**Example.** $\int \underbrace{(7x+8)}_{\text{degree } 1}{}^9 dx = ?$

$$x \curvearrowright \underbrace{7x+8}_{u} \curvearrowright (7x+8)^9$$

Let $u = 7x+8$ then $\frac{du}{dx} = 7 \rightsquigarrow \frac{1}{7} du = dx$

$$= \int u^9 \cdot \frac{1}{7} du = \frac{1}{7} \int u^9 du = \frac{1}{7} \cdot \frac{1}{10} u^{10} + C$$

$$= \frac{1}{70} (7x+8)^{10} + C$$

## Example

$$\int \sqrt{6x+2}\,dx =$$

$= \int (6x+2)^{1/2} dx$ let $u = 6x+2$ } no $dx$ here

$\frac{du}{dx} = 6$ so $\frac{1}{6} du = dx$

$= \int u^{1/2} \cdot \frac{1}{6} du$

$= \frac{1}{6} \int u^{1/2} du = \frac{1}{6} \cdot \frac{1}{\frac{1}{2}+1} u^{1/2+1} + C$

$= \frac{1}{9} u^{3/2} + C = \frac{1}{9} (6x+2)^{3/2} + C$

## More involved examples II

Recall that we don't know $\int \ln(x)dx$. So if there is a natural logarithm in the integral, first try to set $u$ equal to the logarithm.

**Example.** $\int \frac{\ln(x^2)}{x}dx = \; ?$

Let $u = \ln(x^2) = 2\ln(x)$

$$\frac{du}{dx} = 2 \cdot \frac{1}{x} \rightsquigarrow \frac{1}{2}du = \frac{1}{x}dx$$

$$= \int u \cdot \frac{1}{2}du = \frac{1}{2}\int u\,du = \frac{1}{2}\cdot\frac{1}{2}u^2 + C$$

$$= \frac{1}{4}u^2 + C = \frac{1}{4}(2\ln(x))^2 + C = (\ln(x))^2 + C$$

$$\frac{1}{\cancel{4}}\cdot \cancel{2}\ln(x)\cdot\cancel{2}\ln(x) + C$$