McMaster University

FACULTY OF SCIENCE
Department Of Mathematics & Statistics

Section 5.2: Integration by substitution.
Section 5.3: The area under a curve.

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MATH 1MM3 Winter 2023
Lecture 18

Test 2 info

Material covered: (This is a preliminary list, please check childsmath for a final list early next week) Sections 4.3 (including review material for finding largest/smalles values, intervals of increase/decrease, concave up/down), 4.4, 5.1, 5.2

Assignment 5 due Monday, Feb 27

Review session

Time: Feb 25, 2023 , 2-4 PM Eastern Time (US and Canada)
Link: mcmaster.zoom.us/j/95840371928?pwd=Z0x3M3hBeXpRS2Ixako1ZDluUUcwdz09

Meeting ID: 958 4037 1928
Passcode: 307140

The Math Help Centre is available to help during Reading Week, Tuesday Feb. 21 to Fri. Feb 24, ONLINE ONLY in their MSTeams channel. TA times are posted in their General Channel.

Example

Find the antiderivative of $\frac{1}{x\ln(x)}$ passing through the point $(e^5, \ln(10))$.

Soln: $\int \frac{1}{x\,\ln(x)}\,dx =$

Let $u = \ln(x) \rightarrow \frac{du}{dx} = \frac{1}{x}$

$du = \boxed{\frac{1}{x}\,dx}$

$= \int \frac{1}{u}\cdot du = \ln|u| + C$

$= \ln(|\ln(x)|) + C$

To solve for C: $\ln(|\ln(e^5)|) + C = \ln(10)$

$\ln(|5|) + C = \ln(10)$

$C = \ln(10) - \ln(5) = \ln\left(\frac{10}{5}\right) = \ln(2)$

Answer: $\ln(|\ln(x)|) + \ln(2)$

A nontypical example.

$$\int \frac{t-1}{t+1}\,dt =$$

Idea: Degree 1: set denominator to u

Let $u = t+1 \rightsquigarrow \frac{du}{dt} = 1$ so $du = dt$

$= \int \frac{\boxed{(t-1)}}{u}\,du$!!! If $u = t+1$

$t = u-1$

$= \int \frac{u-1-1}{u}\,du = \int \frac{u-2}{u}\,du = \int \left(\frac{u}{u} - \frac{2}{u}\right)du$

$= \int \left(1 - \frac{2}{u}\right)du = u - 2\ln|u| + C$

$= t+1 - 2\ln|t+1| + C$

Dealing with sums/differences

$$\int \left(20 + \frac{2x}{x^2+1}\right) dx$$

$$= \underbrace{\int 20\,dx}_{20x + C} + \underbrace{\int \frac{2x}{x^2+1}\,dx}_{u\text{-sub.}}$$

Let $u = x^2 + 1 \rightsquigarrow du = 2x\,dx$

$$= 20x + C + \int \frac{du}{u}$$

$$= 20x + \ln(|u|) + C$$

$$= 20x + \ln(x^2+1) + C$$

Solving separable differential equations by u-substitution

$$\frac{dy}{dx} = \frac{yx}{x^2+3} \quad \text{if} \quad y = 1 \quad \text{when} \quad x = 1$$

Separate:
Integrate: $\int \frac{dy}{y} = \int \frac{x}{x^2+3}\,dx$ | Let $u = x^2+3$, $\frac{1}{2}du = x\,dx$

$\ln|y| = \int \frac{1}{u}\cdot\frac{1}{2}\,du$

$\ln|y| = \frac{1}{2}\ln|u| + C$

$\ln|y| = \frac{1}{2}\ln(x^2+3) + C$ | $x^2+3>0$

Solve for C: substitute $x=1$ and $y=1$

Solving separable differential equations by u-substitution

$$\ln|1| = \frac{1}{2}\ln(1^2+3)+C$$

$$0 = \frac{1}{2}\ln(4)+C \quad \text{so} \quad C = -\frac{1}{2}\ln(4)$$

$$= -\frac{1}{2}\ln(2^2).$$

$$C = -\frac{1}{2}\cdot 2\ln(2) = -\ln(2)$$

Have:

$$\ln|y| = \frac{1}{2}\ln(x^2+3)-\ln(2)$$

Since $y=1>0$, remove abs. value

$\ln(y) = \frac{1}{2}\ln(x^2+3)-\ln(2)$. Take e to both sides:

$$e^{\ln y} = e^{\frac{1}{2}\ln(x^2+3)-\ln(2)}$$

$$y = \frac{e^{\frac{1}{2}\ln(x^2+3)}}{e^{\ln(2)}} = \frac{\left(e^{\ln(x^2+3)}\right)^{1/2}}{2} \quad \text{(power rule)}$$

$$y = \frac{(x^2+3)^{1/2}}{2}$$

Take a minute to digest!

Take a minute to jot down what you can remember from the examples we've done last time and today. For example, can you remember all the types of functions that would be good bets to choose for u?

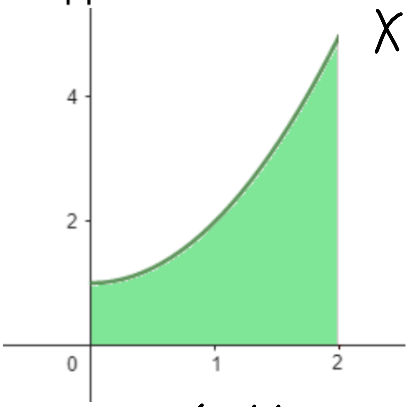
Additional practice problems for integration by u-substitution
(combined examples)
Ex. 3, 5, 21, 23, 25, $\underbrace{29,\ 31}_{!!!}$, 37, 39, 41, 43, 45, 53 in Section 5.2

Section 5.3: The area under a curve

So far, we had an interpretation for the derivative:

The derivative	$\longrightarrow$	the slope of the tangent line
The integral	$\longrightarrow$	????

Example. The paper for an individual candy wrapper has the shape in green: If the cost is 1 cent/cm^2, how much does one wrapper cost?



x^2+1

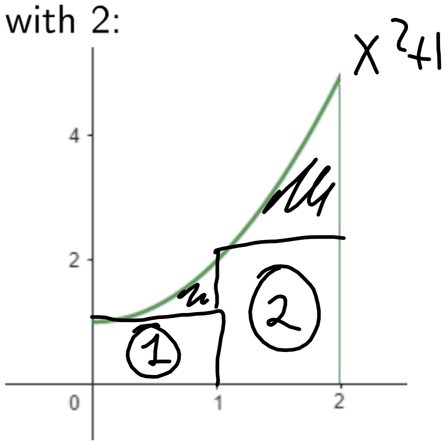
$0 \le x \le 2$

$\text{Cost} = \text{Area}\,(\cancel{cm^2}) \cdot 1\left(\frac{\text{cent}}{\cancel{cm^2}}\right)$

$= \text{Area}\,(\text{cents})$

Approximating the area by smaller pieces (2 pieces)

We have no formula for the area of such a shape, but we can approximate it. We do so by dividing ito smaller pieces. We start with 2:



1. Base length 1
   height $f(0) = 0^2 + 1 = 1$

2. Base length 1
   height $f(1) = 1^2 + 1 = 2$

Total: $1 + 2 = 3$