McMaster University

FACULTY OF SCIENCE
Department Of Mathematics & Statistics

Section 5.3: Evaluating definite integrals

Angelica Babei

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Lecture 20

The fundamental theorem of calculus

Given a function $f(x) \geq 0$ on the interval $[a, b]$, the area under the curve is

$$\text{Area} = \int_a^b f(x)dx.$$

This is called a **definite integral**. This area can also be interpreted as an accumulation of people, objects, money etc., if we are given the rate of change of a function that measures people, objects, money etc.

Theorem (The Fundamental Theorem of Calculus)

If $f(x)$ is continuous on $a \leq x \leq b$, and $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$$

Additional practice problems for Riemann sums and the Fundamental theorem of calculus:
Ex. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 51, 59, 61, 63, 65 in Section 5.3.

Take a minute to digest!

Think of some specific examples, besides the area under a curve, where the definite integral calculates an accumulation. For example, given the growth of a population as a function, the definite integral from a to b of this function gives the total new population between times a and b.

Properties of definite integrals part I

- Constant multiple: $\int_a^b kf(x)dx = k\int_a^b f(x)dx$

$$\int_2^3 2x^3\,dx = 2\int_2^3 x^3\,dx = \cancel{2}\cdot\frac{1}{\cancel{4}_{2}}x^4\Big|_2^3$$

$$=\frac{1}{2}(3^4-2^4) = 32.5$$

- Sum/difference:

$$\int_a^b [f(x)\pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_5^3 (x-e^{2x})\,dx = \int_5^3 x\,dx - \int_5^3 e^{2x}\,dx =$$

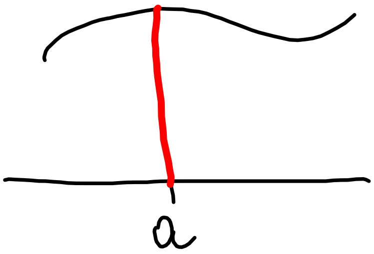
$$\frac{1}{2}x^2\Big|_5^3 - \frac{1}{2}e^{2x}\Big|_5^3 = \frac{1}{2}(3^2-5^2)$$

$$-\frac{1}{2}(e^6-e^{10}) = -8-\frac{1}{2}e^6+\frac{1}{2}e^{10}$$

Properties of definite integrals part II

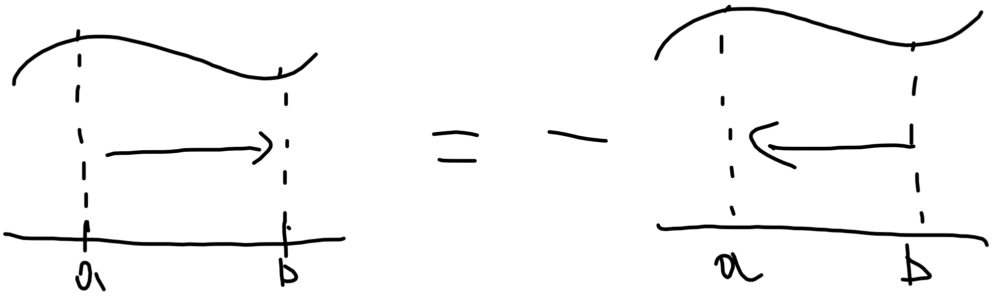
- $\int_a^a f(x)dx = 0$

Area of segment line is 0 (no surface)



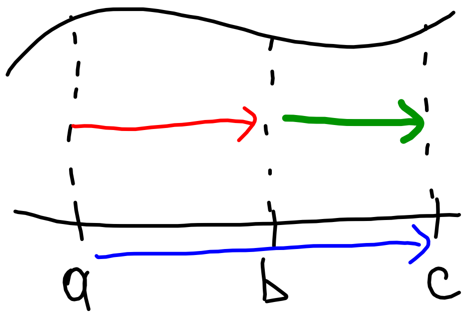
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Definite integrals have a direction.



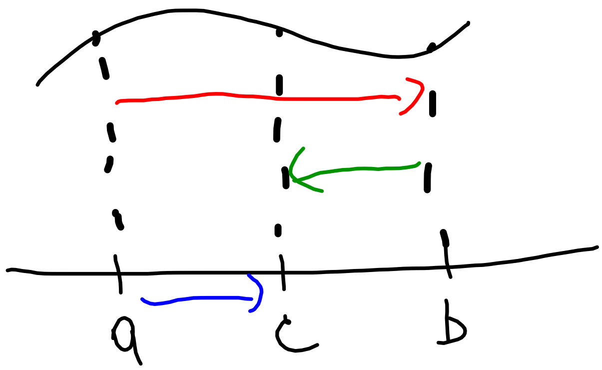
Properties of definite integrals part III

- $$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$ (Subdivision rule)



Rule applies even if a, b, c not in order.

e.g.: $a \le c \le b$



Example

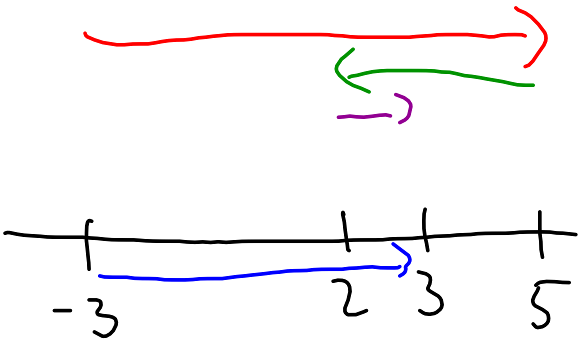
If

$$\underbrace{\int_{-3}^{5}}\, f(x)dx = 16, \quad \underbrace{\int_{2}^{3}}\, f(x)dx = 1, \quad \text{and} \quad \underbrace{\int_{2}^{5}}\, f(x)dx = 10,$$

find

$$\int_{-3}^{3} f(x)dx.$$

$$\int_5^2 = -10$$



$$\int_{-3}^{3} = \int_{-3}^{5} + \int_{5}^{2} + \int_{2}^{3}$$

$$= 16 - 10 + 1$$

$$= 7$$

Example

Flip: $\int_{1}^{-3} = -6$ $\quad \int_{1}^{-3} = -(-4) = 4$

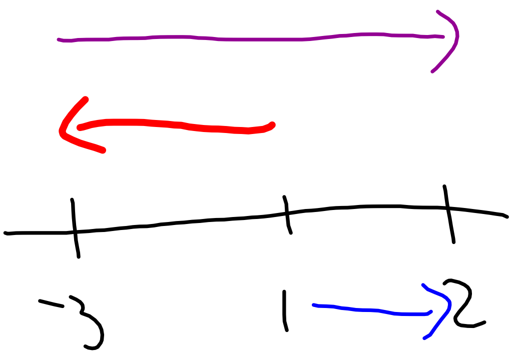
If

$$\int_{-3}^{2} f(x)dx = 5, \quad \int_{-3}^{2} g(x)dx = -2, \quad \int_{-3}^{1} f(x)dx = 6, \quad \int_{-3}^{1} g(x)dx = -4,$$

evaluate

$$\int_{\boxed{1}}^{\boxed{2}} [2f(x) - g(x)]dx.$$

① Find $1 \to 2$



② Evaluate integral