McMaster University

FACULTY OF SCIENCE
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5.3 More definite integrals

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MATH 1MM3 Winter 2023
Lecture 21

Example

Flip $\int_1^{-3} f(x)dx = -6$

If

$$\int_{-3}^{2} f(x)dx = 5,\quad \int_{-3}^{2} g(x)dx = -2,\quad \int_{-3}^{1} f(x)dx = 6,\quad \int_{-3}^{1} g(x)dx = -4,$$

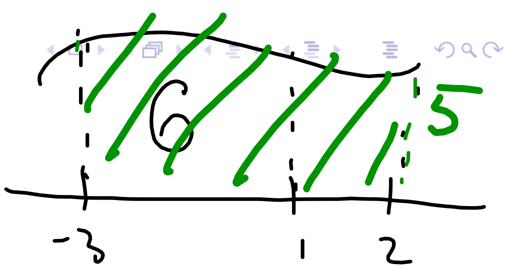
Flip $\int_1^{-3} g(x)dx = -(-4) = 4$

evaluate $$\int_1^2 [2f(x) - g(x)]dx.$$

① Find $1 \to 2$

So $\boxed{\int_1^2 = \int_1^{-3} + \int_{-3}^{2}}$

$$\int_1^2 [2f(x) - g(x)]dx$$
$$= \int_1^2 2f(x)dx - \int_1^2 g(x)dx$$
$$= 2\int_1^2 f(x)dx - \int_1^2 g(x)dx$$



Example (cont.)

Flip $\int_{1}^{-3} f(x)dx = -6$

$$\int_{-3}^{2} f(x)dx = 5,\ \int_{-3}^{2} g(x)dx = -2,\ \int_{-3}^{1} f(x)dx = 6,\ \int_{-3}^{1} g(x)dx = -4,$$

Flip $\int_{1}^{-3} g(x)dx = 4$

evaluate $\int_{1}^{2} [2f(x) - g(x)]dx.$

$$\int_{1}^{2} = \int_{1}^{-3} + \int_{-3}^{2}$$

$$\int_{1}^{2} [2f(x) - g(x)]dx = \boxed{2\int_{1}^{2} f(x)dx - \int_{1}^{2} g(x)dx}$$

$$\int_{1}^{2} f(x)dx = \int_{1}^{-3} f(x)dx + \int_{-3}^{2} f(x)dx = -6 + 5 = -1$$

$$\int_{1}^{2} g(x)dx = \int_{1}^{-3} g(x)dx + \int_{-3}^{2} g(x)dx = 4 + (-2) = 2$$

Combine $\boxed{}$ get $2 \cdot (-1) - 2 = -4$

Negative definite integrals

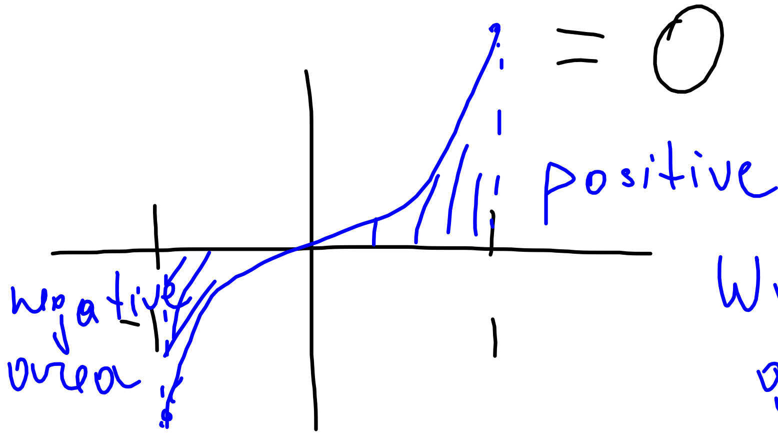
If $f(x) \leq 0$ on an interval $a \leq x \leq b$, then its graph is under the x-axis, and $\int_a^b f(x)dx \leq 0$. So definite integrals have two sense of directions: a horizontal one based on whether we move from left to right or viceversa, as well as a vertical one based on whether the graph is above or below the x-axis.

Example. $\int_{-1}^{1} (e^x - e^{-x})dx$ $= \left(e^x - \frac{1}{-1} e^{-x}\right)\Big|_{-1}^{1}$

$= \left(e^x + e^{-x}\right)\Big|_{-1}^{1} = \left(e^1 + e^{-1}\right)$

$- \left(e^{-1} + e^{-(-1)}\right) = \cancel{e^1} + \cancel{e^{-1}} - \cancel{e^{-1}} - \cancel{e^1}$

$= 0$



Will cancel out, get 0.

Additional practice problems forproperties definite integrals:
Ex. 45, 47, 49 in Section 5.3.

u-substitution for definite integrals (Method 1)

Example. $\int_1^3 \frac{2x}{x^2+1}\,dx = ?$

2 methods:

- Find the antiderivative in terms of x as before, then evaluate it at 1 and 3.

Indefinite

$$\int \frac{2x}{x^2+1}\,dx \underset{\text{slide}}{\overset{\text{next}}{=}} \int \frac{1}{u}\,du = \ln|u| + C$$

$$= \ln(x^2+1) + C$$

Definite: $\int_1^3 \frac{2x}{x^2+1}\,dx = \ln(x^2+1)\Big|_1^3 =$

$$= \ln(3^2+1) - \ln(1^2+1) = \ln(10) - \ln(2)$$
$$= \ln(5)$$

u-substitution for definite integrals (Method 2)

- After doing the substitution $u = g(x)$, change the integration bounds

$$\int_a^b \dots dx \qquad a \longrightarrow \quad c = g(a) \qquad b \longrightarrow \quad d = g(b) \qquad \int_c^d \dots du$$

and evaluate

$$\int_{u=c}^{u=d} \dots d(u).$$

$$\int_{x=1}^{x=3} \frac{2x}{x^2+1}\,dx = \int_{u=2}^{u=10} \frac{1}{u}\,du = \ln|u| \Big|_2^{10} = \ln(10) - \ln(2) = \ln\left(\frac{10}{2}\right) = \ln(5)$$

$$u = x^2+1 \rightsquigarrow \frac{du}{dx} = 2x \rightsquigarrow du = 2x\,dx$$

$$x = 1 \rightsquigarrow 1^2+1 = 2$$

$$x = 3 \rightsquigarrow 3^2+1 = 10$$

Examples

$$\int_{-3}^{0} (2x+6)^4 \, dx$$

Let $u = 2x+6 \rightsquigarrow \frac{1}{2} du = dx$

$$\int_{x=-3}^{x=0} (2x+6)^4 dx = \int_{u=0}^{u=6} u^4 \cdot \frac{1}{2} du$$

$x = -3 \rightsquigarrow u = 2 \cdot (-3) + 6 = 0$

$x = 0 \rightsquigarrow u = 2 \cdot 0 + 6 = 6$

$$\int_0^6 \frac{1}{2} u^4 du = ?$$