McMaster University
FACULTY OF SCIENCE
Department Of Mathematics & Statistics

6.1: Integration by parts
5.5: Continuous income streams

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MATH 1MM3 Winter 2023
Lecture 26

Test 3 info

Material covered: Sections 5.3, 5.4, 5.5, 6.1

Assignment 7 due Monday, Mar 27; please do it before the test.

Review session

Time: Mar 18, 2023 , 2-4 PM Eastern Time (US and Canada)
Link: mcmaster.zoom.us/j/92371889129?pwd=dVdPOWpreDVDbWU4SGQyWGVablhTQT09

Meeting ID: 923 7188 9129
Passcode: 563321

An interesting example

$$\int_8^{160} \boxed{\left(5 - \frac{1}{2}x\right)} \sqrt[3]{\underbrace{7 + \frac{1}{8}x}_{u}}\,dx =$$

Sketch of solution via u-sub:

Let $u = 7 + \frac{1}{8}x$ so $du = \frac{1}{8}dx$.

To handle $\boxed{}$, write $x = 8u - 56$

Bounds:

$x = 160 \rightsquigarrow 7 + \frac{1}{8} \cdot 160 = 27$

$x = 8 \rightsquigarrow 7 + \frac{1}{8} \cdot 8 = 8$

$$\text{integral} = \int_8^{27} \left(5 - \frac{1}{2} \cdot (8u - 56)\right) \cdot \sqrt[3]{u} \cdot 8\,du$$

Example (cont.)

$\int u\,dv = uv - \int v\,du$

$$\int_8^{160} \underbrace{\left(5 - \frac{1}{2}x\right)}_{} \sqrt[3]{7 + \frac{1}{8}x}\,dx =$$

Solution via integration by parts:

Let $u = 5 - \frac{1}{2}x$ so $du = -\frac{1}{2}dx$

$v'(x) = \sqrt[3]{7 + \frac{1}{8}x}$ so $v(x) = \int \underbrace{(7 + \frac{1}{8}x)}_{u\text{-sub.}}{}^{1/3}dx$

Let $w = 7 + \frac{1}{8}x$ so $dw = \frac{1}{8}dx$, and $8\,dw = dx$

So $v(x) = \int w^{1/3} \cdot 8\,dw = \frac{8}{\frac{1}{3}+1} w^{1/3+1} = 6w^{4/3}$

$= 6(7 + \frac{1}{8}x)^{4/3}$

So $\int_8^{160} u\,dv = \underbrace{(5 - \frac{1}{2}x)}_{u} \cdot \underbrace{6(7 + \frac{1}{8}x)^{4/3}}_{v}\Big|_8^{160}$

$- \int_8^{160} \underbrace{6(7 + \frac{1}{8}x)^{4/3}}_{v} \cdot \underbrace{(-\frac{1}{2})dx}_{du}$

Example (cont.)

$$= 6\left(5-\frac{1}{2}x\right)\left(7+\frac{1}{8}x\right)^{4/3}\Big|_{8}^{160} + 3\int_{8}^{160} \underbrace{\left(7+\frac{1}{8}x\right)^{4/3} dx}_{\text{same u-sub:}}$$

$w = 7+\frac{1}{8}x$

$8\,dw = dx$

$$\int \left(7+\frac{1}{8}x\right)^{4/3} dx = \int 8\, w^{4/3}\, dw$$

$$= \frac{8}{\frac{4}{3}+1} w^{4/3+1} = \frac{24}{7} w^{7/3} = \frac{24}{7}\left(7+\frac{1}{8}x\right)^{7/3}$$

$$= 6\left(5-\frac{1}{2}x\right)\left(7+\frac{1}{8}x\right)^{4/3}\Big|_{8}^{160} + \frac{3\cdot 24}{7}\left(7+\frac{1}{8}x\right)^{7/3}\Big|_{8}^{160}$$

Example (cont.)

$$= 6\left(5-\frac{160}{2}\right)\left(7+\frac{160}{8}\right)^{4/3} - 6\left(5-\frac{8}{2}\right)\left(7+\frac{8}{8}\right)^{4/3}$$

$$+\frac{72}{7}\left(7+\frac{1}{8}\cdot 160\right)^{7/3} - \frac{72}{7}\left(7+\frac{1}{8}\cdot 8\right)^{7/3}$$

$$= 6\cdot(-75)\cdot 27^{4/3} - 6\cdot 1\cdot 8^{4/3}$$

$$+\frac{72}{7}\cdot 27^{7/3} - \frac{72}{7}\cdot 8^{7/3}$$

$$\boxed{\begin{aligned} 27 &= 3^3 \\ 8 &= 2^3 \end{aligned}}$$

$$= -450\cdot(3^3)^{4/3} - 6\cdot(2^3)^{4/3}$$

$$+\frac{72}{7}\cdot(3^3)^{7/3} - \frac{72}{7}(2^3)^{7/3}$$

$$= -450\cdot 3^4 - 6\cdot 2^4 + \frac{72}{7}\cdot 3^7 - \frac{72}{7}\cdot 2^7$$

$$= -\frac{107574}{7}$$

u-sub or by parts?

For the integrals in the following 3 slides, decide if the integration can be done via u-substitution, by parts, neither, or either:

1. $\int x^7 \ln(7x) dx$: by parts!

$u = \ln(7x) \rightsquigarrow du = \frac{1}{x} dx$

$v'(x) = x^7$ so $v(x) = \int x^7 dx = \frac{1}{8} x^8$

$$\int u \, dv = \underbrace{\ln(7x)}_{u} \cdot \underbrace{\frac{1}{8} x^8}_{v} - \int \frac{1}{8} x^8 \cdot \frac{1}{x} dx$$

u-sub

2. $\int \frac{1}{x \ln(7x)} dx$

Let $u = \ln(7x) \rightsquigarrow du = \frac{1}{x} dx$

$= \int \frac{1}{u} du$

3. $\int \frac{1}{x^7 \ln(7x)} dx$: neither!

u-sub or by parts?

1. $\int x^3 e^x \, dx$: by parts

$u = x^3 \rightsquigarrow du = 3x^2 dx$

$v'(x) = e^x \rightsquigarrow v = \int e^x dx = e^x$

$\int u \, dv = x^3 \cdot e^x - \boxed{\int e^x \cdot 3x^2 dx} \to$ do by parts again.

2. $\int \underbrace{x^3}\, e^{x^4} \underbrace{dx}$: u-sub.

$u = x^4 \rightsquigarrow \frac{1}{4} du = \underbrace{x^3 dx}$

$= \int \underbrace{\frac{1}{4}} \cdot e^u \underbrace{du}$

3. $\int x^3 e^{x^2} dx$: by parts first, then u-sub.

$= \int x^2 \cdot \underbrace{x e^{x^2} dx}_{\text{u-sub.}}$ | Let $v'(x) = x e^{x^2}$ then $v(x) = \int x e^{x^2} dx$ $= \underbrace{\frac{1}{2} e^{x^2}}$ by u-sub

Let $u = x^2 \rightsquigarrow du = 2x dx$

$\int u \, dv = \underbrace{x^2}_{u} \cdot \underbrace{\frac{1}{2} e^{x^2}}_{v} - \underbrace{\int \frac{1}{\cancel{2}} e^{x^2} \cdot \cancel{2} x \, dx}_{\text{by u-sub.}}$