McMaster University

FACULTY OF SCIENCE
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7.1: Functions of multiple variables (cont.)

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Lecture 29

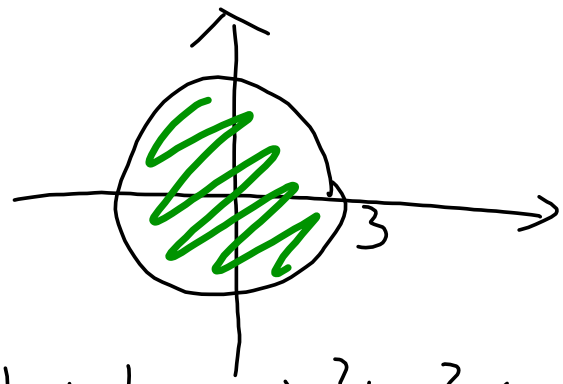
The domain of $f(x, y)$: example I

Find the domain of $f(x,y) = \sqrt{9 - x^2 - y^2}$.

Cannot take $\sqrt{\ }$ of negative numbers

$9 - x^2 - y^2 \geq 0$

$x^2 + y^2 \leq 9$



This region, the pairs (x, y) where $x^2 + y^2 \leq 9$ are the points on or inside the circle $(x^2 + y^2 = 3^2)$ of radius 3 centered at the origin.

The domain of $f(x, y)$: example II

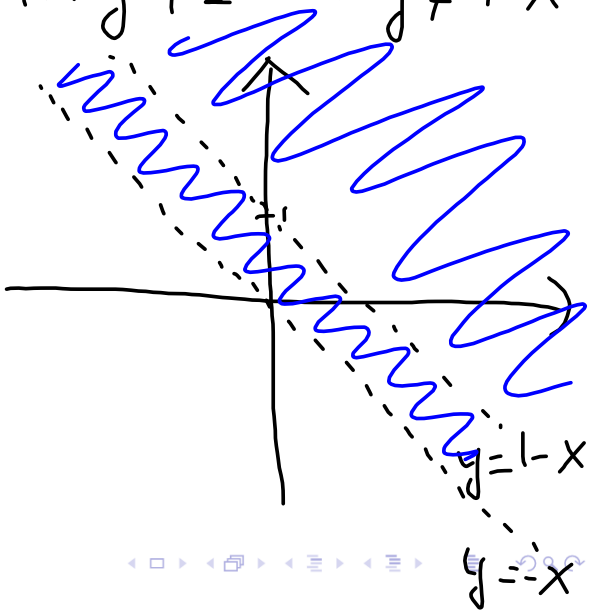
Find the domain of $f(x, y) = \dfrac{x}{\ln(x + y)}$.

2 conditions: ① Cannot divide by 0

so $\ln(x+y) \neq 0 \xrightarrow{\text{so}} x+y \neq 1 \rightarrow y \neq 1-x$

② Can only take ln of positive numbers, so

$x+y > 0 \xrightarrow{\text{so}} \underline{y > -x}$



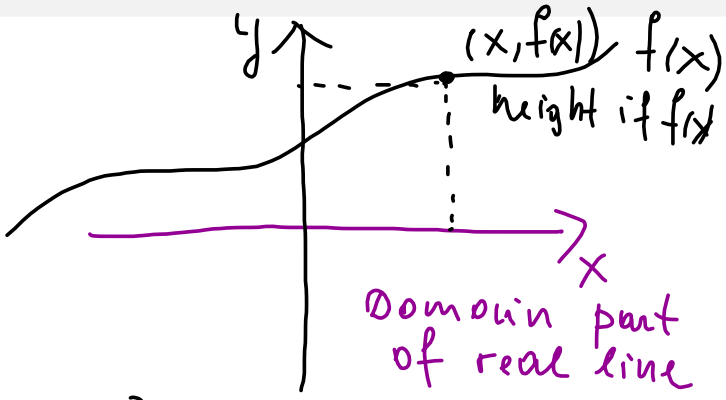
So these are the points (x, y) strictly above the line $y = -x$

To conclude, the domain is the region above $y = -x$, without $y = 1-x$.

Graphs of functions of 2 variables: think in 3D!

Before: $y = f(x)$

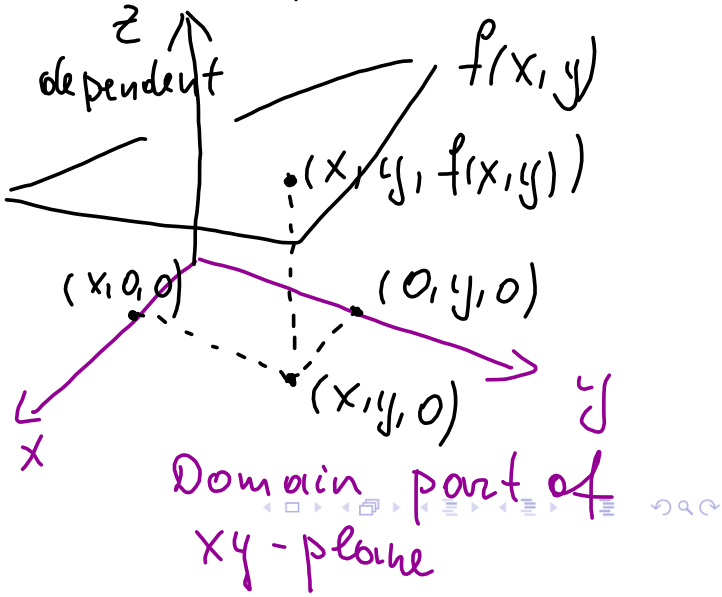
Graphs are curves



Now: $z = f(x,y)$

height is $f(x,y)$

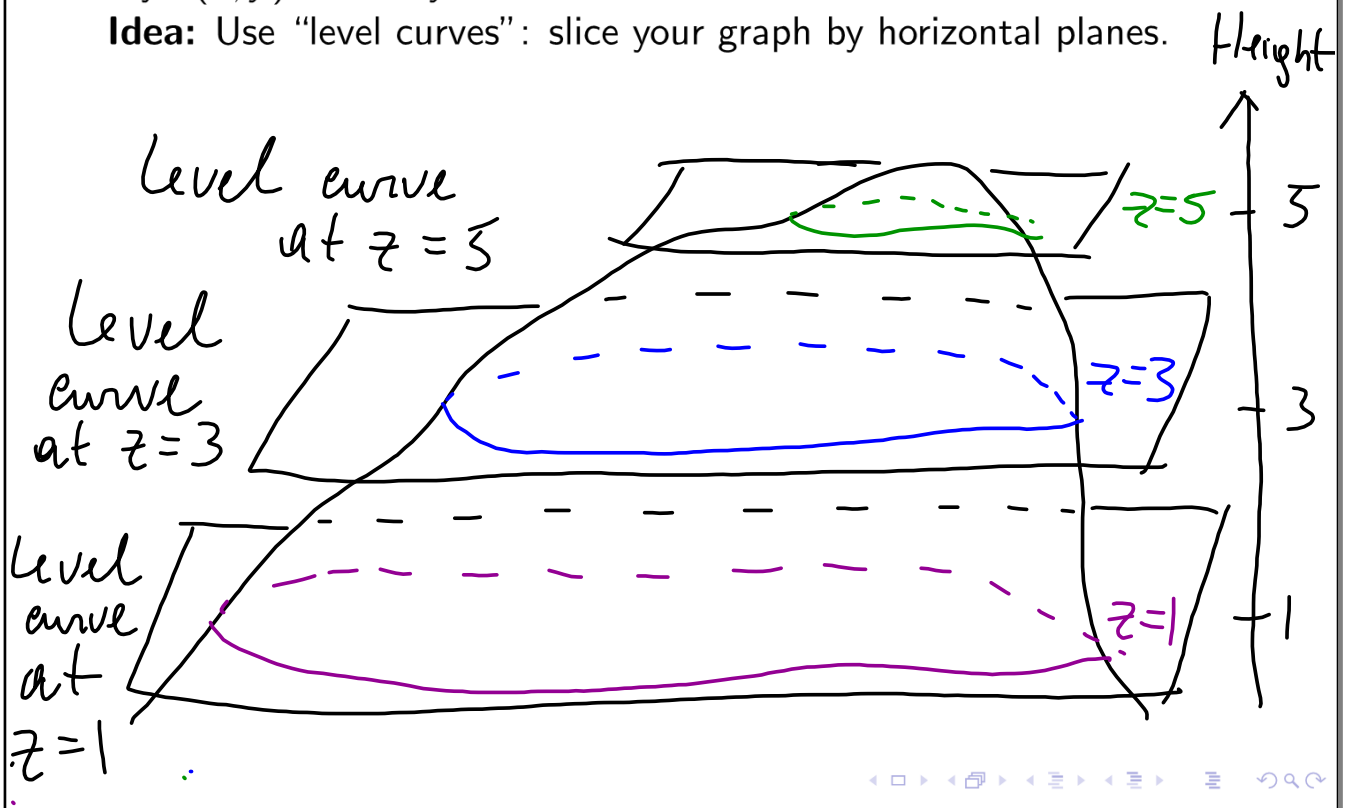
Graph is 2-dim sheet



Graphs of functions of 2 variables via level curves

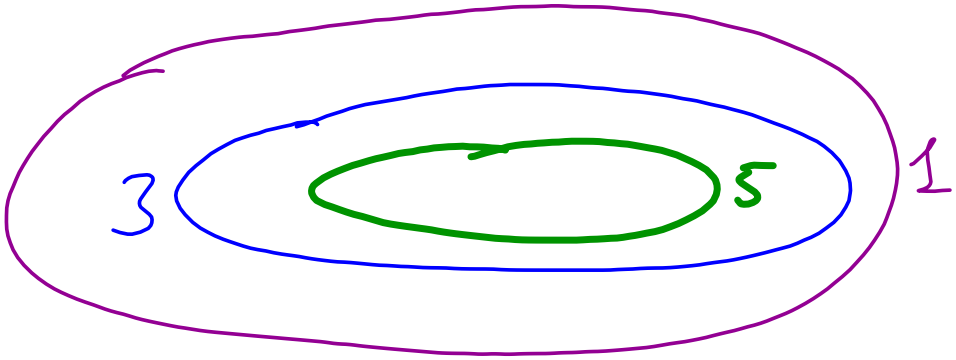
Question: How can we sketch the graph of a particular function, say $f(x, y) = x^2 + y^2$?

Idea: Use "level curves": slice your graph by horizontal planes.



Level curves

We often represent the level curves in 2D:



The intersection of the plane $z = h$ with the graph $f(x, y)$ is called the "the level curve of $f(x, y)$ at $z = h$", described by the equation $f(x, y) = h$.

For a cool interactive page on level curves, **check this out**!

Level curves: Example I

$f(x,y) = h$

Based on level curves of $f(x, y) = x^2 + y^2$, what does its graph look like?

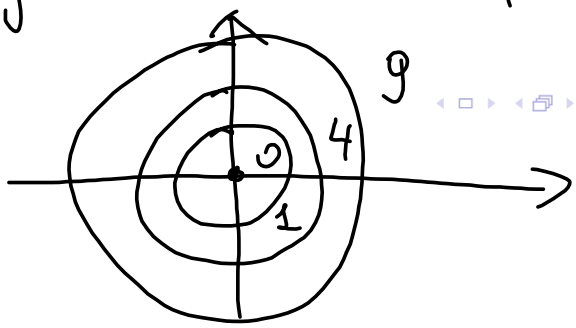
If $h < 0$: $x^2 + y^2 = h < 0$ has no solution so graph is entirely above the xy-plane.

$h = 0$: $x^2 + y^2 = 0$: point $(0,0)$ only solution

$h = 1$: $x^2 + y^2 = 1$: circle of radius 1 at origin

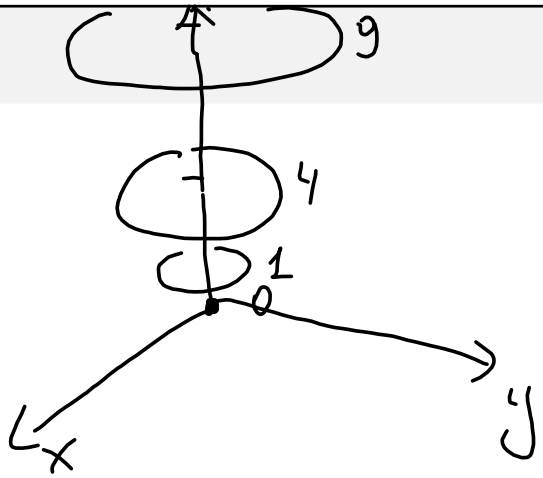
$h = 4$: $x^2 + y^2 = 4 = 2^2$: circle of radius 2

$h = 9$: $x^2 + y^2 = 3^2$: circle of radius 3



Example I (cont.)

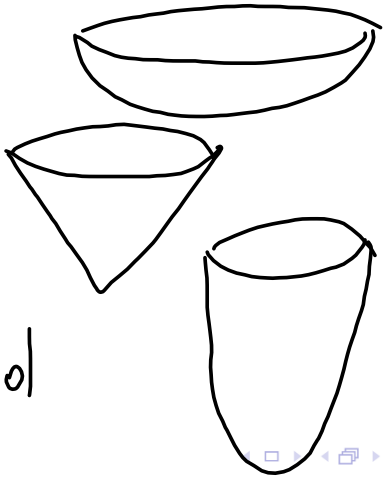
In 3D,



3 options: bowl

cone

paraboloid



$f(x,y) = x^2 + y^2$

Example II

Sketch the level curves $z = -2, -1, 0, 1, 2$ for $f(x, y) = \frac{x}{y}$.

Solve $f(x,y) = z$ for $z = -2, -1, 0, 1, 2$

$z = -2$: $\frac{x}{y} = -2$ so $y = -\frac{1}{2}x$

$z = -1$: $\frac{x}{y} = -1$ so $y = -x$

$z = 0$: $\frac{x}{y} = 0$ so $x = 0$

$z = 1$: $\frac{x}{y} = 1$ so $y = x$

$z = 2$: so $\frac{x}{y} = 2$ so $y = \frac{1}{2}x$